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Optimum Design of Cyclone Separator

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Cyclone separators are one of the most widely used gas-solid separators. Although the optimal design of cyclone separators has been suggested earlier, the earlier works do not include all the critical parameters responsible for minimizing the pressure drop which is quite decisive to obtain a correct optimal design. In this article, the optimal design of the cyclone separator has been formulated as a geometric programming with a single degree of difficulty. The solution of the problem yields the optimum values of the number of cyclones to be used in parallel, and the inside diameter of cyclone shell and exit pipe, when a specified flow rate of gas is to be separated from solid particles, when the cut diameter is already specified. © 2009 American Institute of Chemical Engineers *AIChE J, 55: 2279–2283, 2009*

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Introduction

Cyclone separators are one of the most widely used gas-solid separators. Cyclones are frequently used as final collectors where large particles are to be caught. Efficiency is generally good for dusts where particles are larger than about 5 μm in diameter.[1] They can also be used as pre-cleaners for a more efficient collector such as an electrostatic precipitator, scrubber or fabric filter. The major costs involved in running a number of parallel cyclone separators are the fixed costs and the operating costs calculated per unit time. The fixed costs decrease if the number of cyclones is increased; however this in turn increases the pressure drop, thus increasing the pumping costs. The pressure drop in a cyclone separator can also be decreased or increased by varying the inside diameter of the cyclone shell or that of the exit pipe. But again, both these parameters have opposite effects on the total costs of the cyclone and therefore, an optimal choice of these parameters is entailed.

For an accurate optimal design of a cyclone, it is quite necessary to use a reliable pressure drop equation for it. Over the years, a number of equations have been proposed to predict cyclone pressure drop and efficiency.[1] Optimization has also been tried[3,4] using the design method proposed by Coulson and Richardson,[2] however such procedures lack the consideration of some of the key parameters in the optimum design of a cyclone separator, viz. the diameter of the exit pipe. See Figure 1. Thus, an optimal design using these proposed designs is deficient in optimizing the pressure drop due to the losses at exit, which can substantially affect the final optimum design. In this article, we consider an exhaustive equation for pressure drop applicable for Stairmand type high efficiency cyclones, as suggested by Stairmand,[5] and optimize the design taking into consideration the inside diameters of the cyclone shell and the exit pipe. The cost function of the cyclone separator has been formulated as a geometric programming with a single degree of difficulty. It is assumed that the flow rate of the gas and the cut diameter of the particles to be separated are known for the specific design of the cyclone separator. The idea presented in this article can be subsequently used for any type of cyclones by changing the pressure drop equation used.

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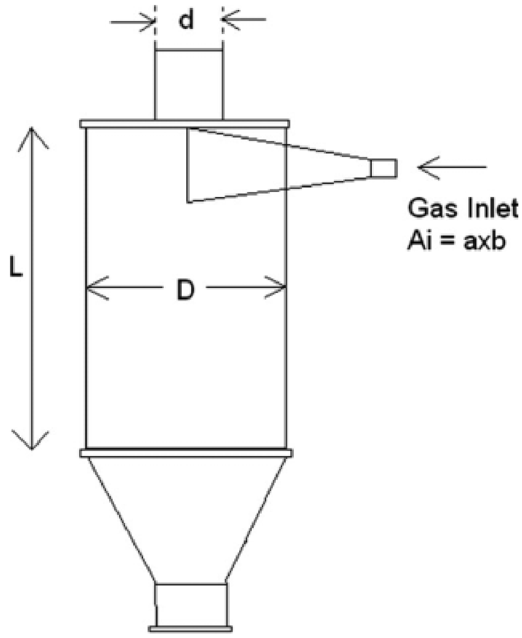

Figure 1. Stairmand type high efficiency cyclone separator.

Cost Function

The cost of the cyclone separator system F_1 is

$$F_1 = C_{\text{power}} + C_{\text{capital}} \tag{1}$$

where C_{power} is the cost involved in pumping the fluid and C_{capital} is the capital cost given by

$$C_{\text{capital}} = k_m D^m + k_m d^m \tag{2}$$

where D = inside diameter of cyclone shell; and d = inside diameter of exit pipe. The constants k_m, and m can be obtained from the prices of cyclones of different diameters. The capitalized cost of power is given by

$$C_{\text{power}} = k_T Q \Delta p \tag{3}$$

where Q = volumetric flow rate through the cyclone; k_T = coefficient given by

$$k_T = \frac{8.76 F_A F_D r_e}{r} \tag{4}$$

wherein F_A and F_D = the annual and daily averaging factors respectively; r_e = rate of electricity per kWatt-hour, r = interest rate expressed as a fraction; and Δp = pressure drop in cyclone separator written as[5]

$$\Delta p = \frac{\rho}{2}\left[V_I^2\left\{1 + 2\Phi^2\left(\frac{2R_I}{R_E} - 1\right)\right\} + 2V_E^2\right] \tag{5}$$

where V_I and V_E = linear speed in inlet and outlet ducts respectively; Φ = velocity ratio U_1/V_1 given by

$$\Phi = \frac{A_i}{2fA_s}\sqrt{\frac{R_E}{2R_I}}\left(\sqrt{1 + \frac{8fA_sR_I}{A_iR_E}} - 1\right) \tag{6}$$

where U_1 = spinning speed at mean inlet radius; A_i = area of cyclone inlet given by $A_i = ab$ wherein a = height of inlet slot and b = width of inlet slot; A_s = surface area of cyclone exposed to the spinning fluid that can be sufficiently accurately obtained by Stairmand[5]

$$A_s = \pi DL + 0.5\pi D^2 + \pi dl \tag{7}$$

wherein L is the length of the cyclone; l = the length of exit pipe, f = friction factor that can be taken as 0.005; R_E radius of exit pipe = $0.5d$, and R_I = mean inlet radius given by

$$R_I = 0.5(D - b) \tag{8}$$

In practice, however, the value of Φ is not affected greatly by relatively small changes in the surface area of the exit pipe and it is sufficiently accurate to neglect the third term in Eq. 7. Further, the linear speeds are expressed as

$$V_I = \frac{Q}{A_i N} \tag{9}$$

$$V_E = \frac{4Q}{\pi d^2 N} \tag{10}$$

where N = number of cyclone separators. Also, the diameter of the cut point can be written as given by Gerrard and Liddle[3]

$$d_{\text{cut}} = \left[\frac{3.6\mu A_i^2 N d}{\pi LDQ\rho(s-1)}\right]^{\frac{1}{2}} \tag{11}$$

where μ = dynamic viscosity the fluid; and ρ = density of the solid particles, and s = density ratio of the solid particles and the fluid. Writing the height and the width of the inlet slot, and the length of the cyclone in terms of the cyclone diameter,[4] we have

$$A_i = a_0 D \times b_0 D = a_0 b_0 D^2 \tag{12}$$

$$L = L_0 D \tag{13}$$

Stairmand[6] recommends the value of a_0, b_0 and L_0 as 0.5, 0.2, and 4 respectively, for high efficiency cyclones; however these values can be different for different practical cases. Using Eq. 12 and 13 in Eq. 9 and 11, we have,

$$V_I = \frac{Q}{a_0 b_0 D^2 N} \tag{14}$$

$$d_{\text{cut}} = \left[\frac{3.6\mu a_0^2 b_0^2 D^2 N d}{\pi L_0 Q\rho(s-1)}\right]^{\frac{1}{2}} \tag{15}$$

Using Eqs. 3, 5, 9, 10, and 14, the capitalized cost of power is

$$C_{\text{power}} = \frac{\rho}{2}k_T Q\left[\left(\frac{Q}{a_0 b_0 D^2 N}\right)^2\left(1 + 4\Phi^2\frac{D - b - 0.5d}{d}\right) + 32\left(\frac{Q}{\pi d^2 N}\right)^2\right] \tag{16}$$

Considering the optimization F_1 of this whole system for three unknowns, viz. D, d and N, we have from Eqs. 1, 2, and 16,

$$F_1 = \frac{\rho}{2} k_T Q \left[\left(\frac{Q}{a_0 b_0 D^2 N} \right)^2 \left(1 + 4\Phi^2 \frac{D - b - 0.5}{d} \right) + 32 \left(\frac{Q}{\pi d^2 N} \right)^2 \right] + k_m D^m N + k_m d^m N \quad (17)$$

Using Eq. 15, writing N in terms of the unknown variables D and d, we have

$$N = \frac{\pi L_0 Q \rho (s-1) d_{cut}^2}{3.6 \mu a_0^2 b_0^2 D^2 d} \quad (18)$$

Using Eq. 18 in Eq. 17 and writing F_1 in terms of only D and d, we have

$$F_1 = \frac{\rho}{2} k_T Q \left[\left(\frac{3.6 \mu a_0 b_0 d}{\pi L_0 \rho (s-1) d_{cut}^2} \right)^2 \left(1 + 4\Phi^2 \frac{D - b - 0.5d}{d} \right) + 32 \left(\frac{3.6 \mu a_0^2 b_0^2 D^2}{\pi^2 L_0 \rho (s-1) d_{cut}^2 d} \right)^2 \right] + \frac{k_m D^{m-2} \pi L_0 Q \rho (s-1) d_{cut}^2}{3.6 \mu a_0^2 b_0^2 d} + \frac{k_m d^{m-1} \pi L_0 Q \rho (s-1) d_{cut}^2}{3.6 \mu a_0^2 b_0^2 D^2} \quad (19)$$

Optimization

Writing F_1 in terms of the variables to be determined in an optimization process

$$F_1 = Ad^2 + \frac{BD^4}{d^2} + \frac{CD^{m-2}}{d} + \frac{Cd^{m-1}}{D^2} \quad (20)$$

where

$$A = 6.48 k_T Q (1 + 4\Phi^2 y) \frac{\mu^2 (a_0 b_0)^2}{\pi^2 L_0^2 \rho (s-1)^2 d_{cut}^4} \quad (21)$$

$$B = \frac{207.36 \mu^2 a_0^4 b_0^4 k_T Q}{\pi^4 L_0^2 \rho (s-1)^2 d_{cut}^4} \quad (22)$$

$$C = k_m \frac{\pi L_0 Q \rho (s-1) d_{cut}^2}{3.6 \mu a_0^2 b_0^2} \quad (23)$$

$$y = \frac{D - b - 0.5d}{d} \quad (24)$$

Equation 20 is in the form of a posynomial (positive polynomial). Thus the minimization of Eq. 20 boils down to a geometric programming problem with a single degree of difficulty.[7] The contributions of various terms of Eq. 20 are defined by the weights w_1, w_2, w_3, and w_4[8] given by

$$w_1 = Ad^2 F_1^{-1} \quad (25)$$

$$w_2 = BD^4 d^{-2} F_1^{-1} \quad (26)$$

$$w_3 = CD^{m-2} d^{-1} F_1^{-1} \quad (27)$$

$$w_4 = CD^{-2} d^{m-1} F_1^{-1} \quad (28)$$

The dual objective function F_2 of Eq. 20 is written as

$$F_2 = \left(\frac{Ad^2}{w_1} \right)^{w_1} \left(\frac{BD^4}{d^2 w_2} \right)^{w_2} \left(\frac{CD^{m-2}}{d w_3} \right)^{w_3} \left(\frac{Cd^{m-1}}{D^2 w_4} \right)^{w_4} \quad (29)$$

The orthogonality condition of (20) for D and d are

$$D : 4w_2^* + (m-2) w_3^* - 2w_4^* = 0 \quad (30)$$

$$d : 2w_1^* - 2w_2^* - w_3^* + (m-1) w_4^* = 0 \quad (31)$$

whereas the normality condition of Eq. 20 is

$$w_1^* + w_2^* + w_3^* + w_4^* = 1 \quad (32)$$

where * corresponds to optimality. Solving Eqs. 30–32 for w_1^*, w_2^*, and w_3^*, one gets

$$w_1^* = \frac{4-m}{2(5-m)} - \frac{m w_4^*}{4} \quad (33)$$

$$w_2^* = \frac{2-m}{2(5-m)} + \frac{m w_4^*}{4} \quad (34)$$

$$w_3^* = \frac{2}{5-m} - w_4^* \quad (35)$$

Substituting Eqs. 33–35 in Eq. 28, and simplifying, the optimal dual F_2^* is

$$F_2^* = \left[\frac{4(5-m)A}{2(4-m) - m(5-m) w_4^*} \right]^{\frac{4-m}{2(5-m)}} \times \left[\frac{4(5-m)B}{2(2-m) + m(5-m) w_4^*} \right]^{\frac{2-m}{2(5-m)}} \left[\frac{(5-m)C}{2-(5-m) w_4^*} \right]^{\frac{2}{5-m}} \times \left[\left(\frac{B}{A} \right)^{\frac{m}{4}} \left\{ \frac{2(4-m) - m(5-m) w_4^*}{2(2-m) + m(5-m) w_4^*} \right\}^{\frac{m}{4}} \frac{2-(5-m) w_4^*}{(5-m) w_4^*} \right]^{w_4^*} \quad (36)$$

The dual function given by Eq. 36 depends on the optimal weight w_4^*, which can be obtained by differentiating Eq. 36 with respect to w_4^*, equating it to zero, and simplifying. Considering the complexity of Eq. 36, it can be seen that this approach is prohibitive. Alternatively, equating the factor having exponent w_4^*, on right hand side of Eq. 36 to unity the optimality condition is obtained as,[9]

$$\left(\frac{B}{A} \right)^{\frac{m}{4}} \left\{ \frac{2(4-m) - m(5-m) w_4^*}{2(2-m) + m(5-m) w_4^*} \right\}^{\frac{m}{4}} \frac{2-(5-m) w_4^*}{(5-m) w_4^*} = 1 \quad (37)$$

Using Eqs. 21 and 22, Eq. 37 is written as

$$M = \left[\frac{2(4-m) - m(5-m) w_4^*}{2(2-m) + m(5-m) w_4^*} \right]^{\frac{m}{4}} \frac{2-(5-m) w_4^*}{(5-m) w_4^*} \quad (38)$$

where M is a parameter given by

$$M = \left[\frac{\pi^2}{32}\frac{1+4\Phi^2 y}{(a_0 b_0)^2}\right]^{\frac{m}{4}} \tag{39}$$

Equation 38 is an implicit equation in w_4^*. For all practical applications M lies between 1 and 50. For this range of M, Eq. 38 is fitted to the following explicit form in w_4^*:

$$w_4^* = \frac{2}{5-m}\left(1+\frac{1.617}{1+m}M^{1+0.0221m^{2.5}}\right)^{-1} \tag{40}$$

Using Eqs. 36 and 40, the maximum of the dual F_2^* is obtained. As the maximum of the dual is equal to the minimum of the primal, i.e., $F_1^* = F_2^*$, the optimal cost is obtained as

$$F_1^* = \left[\frac{4(5-m)A}{2(4-m)-m(5-m)w_4^*}\right]^{\frac{4-m}{2(5-m)}} \times \left[\frac{4(5-m)B}{2(2-m)+m(5-m)w_4^*}\right]^{\frac{2-m}{2(5-m)}}\left[\frac{(5-m)C}{2-(5-m)w_4^*}\right]^{\frac{2}{5-m}} \tag{41}$$

Combining Eqs. 24, 32, and 40 the optimal flow rate d^* is

$$d^* = \left[\frac{4(5-m)A}{2(4-m)-m(5-m)w_4^*}\right]^{\frac{m-6}{4(5-m)}} \times \left[\frac{4(5-m)B}{2(2-m)+m(5-m)w_4^*}\right]^{\frac{2-m}{4(5-m)}}\left[\frac{(5-m)C}{2-(5-m)w_4^*}\right]^{\frac{1}{5-m}} \tag{42}$$

Similarly, using Eqs. 25, 26, 34, and 41 the optimal flow rate D^* is

$$D^* = \left[\frac{4(5-m)A}{2(4-m)-m(5-m)w_4^*}\right]^{\frac{-1}{4(5-m)}} \times \left[\frac{4(5-m)B}{2(2-m)+m(5-m)w_4^*}\right]^{\frac{-3}{4(5-m)}}\left[\frac{(5-m)C}{2-(5-m)w_4^*}\right]^{\frac{1}{(5-m)}} \tag{43}$$

The proceeding development is based on the assumption that y and Φ are constants. The variation in these variables can be considered by an iterative procedure. The following procedure is carried out to calculate the variables y and Φ iteratively:

1. Assume a value of D and d.
2. Find y and Φ by using Eq. 6 and 24.
3. Find M using Eq. 39
4. Find w_4^* using Eq. 40
5. Find A, B and C using Eq. 21–23.
6. Find d^* using Eq. 42
7. Find D^* using Eq. 43
8. Repeat steps 2–7 till two successive y and Φ values are close.
9. Reduce d^* and D^* to the nearest commercially available size.
10. Find the actual value of y and Φ by using Eqs. 6 and 24.
11. Find the corrected value of A using Eq. 21.
12. Find F_1 by using (20).
13. Find the actual values of w_1^*, w_2^*, w_3^*, and w_4^* using Eqs. 25–28.

Table 1. Design Iterations

Iteration Number (1)	w_4^* (2)	D (m) (3)	d (kg/s) (4)	y (5)	Φ (6)
0.	–	0.2032	0.0635	–	–
1.	0.1220	0.7169	0.2216	2.0886	0.9056
2.	0.1217	0.7161	0.2205	2.0879	0.9070
3.	0.1216	0.7158	0.2202	2.0877	0.9074
4.	0.1216	0.7157	0.2201	2.0876	0.9075
5.	0.1216	0.7157	0.2201	2.0876	0.9076
6.	0.1216	0.7157	0.2201	2.0876	0.9076

Practical Example

Design a cyclone separator to separate solids from gas flowing at a rate of $Q = 1.0$ m^3/s. The densities of solid and gas are 2000 kg/m^3 and 1.5 kg/m^3, respectively. The viscosity of the gas is given as 0.000025 kg/m s. The cut diameter has been specified as 0.0000046 m. The values of the constants associated with the capital cost and pumping costs are $k_m = 480$ and $k_t = 9.1$ units.

For starting the algorithm, the initial values of outer diameter of inner and outer pipes were assumed to be 0.2032 m and 0.0635 m respectively. Therefore, the initial value of y and Φ are 2.06 and 0.906 respectively. The iterations were carried out till two successive values of y and Φ were obtained. These iterations are shown in Table 1. The values of the inner diameters of the cyclone shell and exit pipe are finally obtained as 0.72 m and 0.22 m, respectively. Thus the commercially available pipes of inner diameter 0.70 m and 0.20 m can be provided. The tube diameters are discontinuous variables due to standardization. After adopting the commercially available diameters the actual value of the weights were found and the optimum value of the number of cyclones was calculated by Eq. 18. Therefore, the number of cyclones that should be used is 4. The final value of the weights w_1^*, w_2^*, w_3^*, and w_4^* comes out to be 0.3489, 0.1597, 0.3698, and 0.1216. Using Eq. 19 it can be seen that the contribution of the capital cost is around 49%, nearly half of the total costs involved.

Salient Points

The diameter of the cyclone shell is around three times the diameter of the exit pipe. The value of the weight w_3^* is maximum, which implies that the capital cost for the cyclone diameter accounts for the maximum cost for such operations. The weight w_1^* is more than two times greater than w_2^*, which clearly indicates that the pressure drop in the cyclone shell is much more than that in the exit pipe. However, it also concludes that the pressure drop in the exit pipe is not small enough to be neglected and contributes around 16% of the total costs involved in running the cyclone separator. A perusal of Eq. 38 reveals that as M varies between 0 and ∞, and Eqs. 33–35 indicate that for $w_4^* = 0$ the optimal weights w_1^*, w_2^*, and w_2^* are positive. Further, Eq. 35 shows that for $w_4^* = 2/(5-m)$ the optimal weight $w_3^* = 0$. Thus the optimal weights have the following ranges:

$$\frac{2-m}{5-m} \le w_1^* \le \frac{4-m}{2(5-m)} \tag{43}$$

$$\frac{2-m}{2(5-m)} \le w_2^* \le \frac{1}{5-m} \tag{44}$$

$$0 \le w_3^* \le \frac{2}{5-m} \tag{45}$$

$$0 \le w_4^* \le \frac{2}{5-m} \tag{46}$$

These ranges indicate contributions of different terms in the optimal objective function. The most dominant is the third term. This implies that the capital cost involved in using a cyclone separator can contribute to a larger percentage of the total costs. In the optimal solution it can be seen that the least significant terms are the second and the fourth term which represent the pressure drop in the exit pipe and the capital cost associated with the exit pipe, respectively. However, contribution of these terms though minimum, but still is not small enough to be neglected completely.

Conclusion

It has been possible to formulate the optimal design of a cyclone separator as a geometric programming problem having single degree of difficulty. Since the optimal design minimizes the sum of the pumping cost and the capital cost associated with not only the shell diameter but also the exit pipe, it is more accurate and gives better cost efficiency for running the cyclone.

Notation

A = cost function coefficient
a = inlet height
a_0 = coefficient
A_i = area of cyclone inlet
A_s = surface area of cyclone exposed to the spinning fluid
B = cost function coefficient
b = inlet width
b_0 = coefficient
C = cost function coefficient
$C_{capital}$ = the capital cost
C_{power} = pumping cost
D = inside diameter of cyclone shell
d = inside diameter of exit pipe
d_{cut} = diameter of the cut point
F_1 = cost function
F_A = annual averaging factor
F_D = daily averaging factor
f = friction factor
k_m = pipe cost coefficient
k_T = pumping cost coefficient
L = length of the cyclone
L_0 = coefficient
m = pipe cost exponent
N = number of cyclone separators
Q = volumetric flow rate through the cyclone
r = interest rate expressed as a fraction
r_e = rate of electricity per kWatt-hour
R_E = radius of exit pipe
R_I = mean inlet radius
s = the density ratio of solid particles and fluid
U_1 = spinning speed at mean inlet radius
V_E = linear speed in outlet duct
V_I = linear speed in inlet duct
W = width of inlet slot
w_1, w_2, w_3, w_4 = weights
Δp = pressure drop
ρ = mass density of fluid
Φ = velocity ratio U_1/V_1
μ = dynamic viscosity of the fluid

Superscript

$*$ = optimal

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